

1. a) Find the equation of the circle which touches the x-axis at the point A(-4,0) and which passes through the point B(0,2).  
b) A second circle of radius 10 units touches the first circle externally at B. Deduce the equation of this second circle.  
(6,4)
2. A circle, centre C, has equation  $x^2 + y^2 - 4x + 2y - 95 = 0$ .  
i) Write down the coordinates of C.  
ii) Chord PQ of this circle has equation  $x - 2y + 6 = 0$ . Calculate the coordinates of the points P and Q.  
iii) Find the equation of the circle formed by reflecting the original circle in the line PQ.  
(1,5,4)
3. The line with equation  $4x + 3y + 7 = 0$  is a tangent at the point P to the circle with centre C(3,2).  
a) Find the equation of the radius CP.  
b) Find the coordinates of P.  
c) Establish the equation of the circle.  
(4,3,3)
4. A circle, centre C, has equation  $x^2 + y^2 - 6x + 2y - 10 = 0$ .  
a) Find the equation of the tangent at the point A(5,3) on this circle.  
b) Show that the line through the point P(0,3) at right angles to this tangent has equation  $y = 2x + 3$  and show that this line is also a tangent to the circle.  
(4,6)
5. a) Find the centre and radius of each of the circles  
 $x^2 + y^2 - 18x + 10y + 6 = 0$   
and  $x^2 + y^2 + 6x - 8y = 0$ .  
b) Hence, or otherwise, show that the circles touch externally.  
c) Use the ratio of the radii to calculate the coordinates of the point of contact.  
d) Obtain the equation of the common tangent at the point of contact.  
(4,4,4,4)

6. A circle with centre  $C$  has equation  $x^2 + y^2 - 2x - 4y - 3 = 0$ .  
The point  $A(3,4)$  is a point on the circumference.

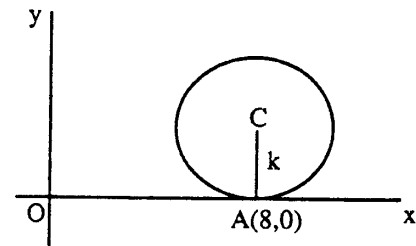
- Find the equation of the tangent to the circle at  $A$ .
- $P$  is the point where the tangent cuts the  $x$ -axis. Find the equation of the circle which passes through the points  $A$ ,  $C$  and  $P$ .

(5,5)

7. a) Show that the equation of the circle which passes through the points  $(-2,6)$ ,  $(4,8)$  and  $(6,6)$  is  $x^2 + y^2 - 4x - 8y = 0$ .
- b) Prove that the line with equation  $2x - y = 10$  is a tangent to this circle and state the coordinates of the point of contact.

(5,5)

8. In the coordinate model shown a circle with radius  $k$  units touches the  $x$ -axis at the point  $A(8,0)$ .



- Show that the equation of the circle is given by  $(x - 8)^2 + (y - k)^2 = k^2$ .
- If  $B(5,9)$  lies on this circle, find  $k$ .
- Establish the equation of the tangent to the circle at  $B$ .

(4,2,4)

9. a) Given that  $A$  and  $B$  are the points  $(4,-1)$  and  $(-2,7)$  respectively, find the equation of the circle with  $AB$  as diameter.
- b) Verify that  $P(-4,3)$  and  $Q(-\frac{2}{5}, -\frac{9}{5})$  are the ends of a chord of the circle parallel to  $AB$ .
- c) Calculate the distance between the chord  $PQ$  and the diameter  $AB$ .

(4,4,4)

1. a) State the centre and the radius of the circle with equation

$$x^2 + y^2 + 2x - 4y - 20 = 0.$$

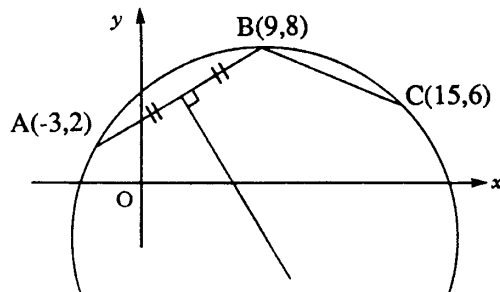
- b) Find the points of intersection P and Q of the line  $x + y = 6$  and the given circle, and write down the coordinates of M, the mid-point of PQ.

- c) A second circle has the same centre as the given circle and has PQ as a tangent. Write down the equation of this second circle

(2,6,3)

2. In the diagram shown, A is the point  $(-3,2)$ , B is the point  $(9,8)$  and C is  $(15,6)$ .

- a) Show that the equation of the perpendicular bisector of AB is  $y + 2x = 11$ .
- b) Find the equation of the perpendicular bisector of BC.
- c) Find the centre and the equation of the circle which passes through the points A, B and C.



(4,4,4)

3. Find the equation of the circle passing through the points  $(0,10)$ ,  $(0,2)$  and  $(3,1)$ . Show that this circle touches the line  $y = 1$  at the point  $(3,1)$ .

Find the coordinates of the other extremity of the diameter through  $(0,2)$  and find the equation of the tangent to the circle at this point.

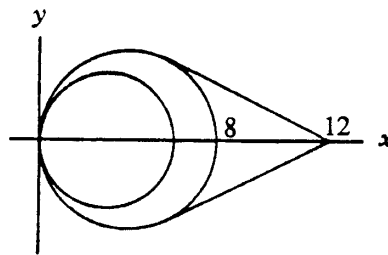
(5,3,2,4)

4. a) Prove that the x-axis is a tangent to the circle  

$$x^2 + y^2 - 10x - 2y + 25 = 0$$
- b) Find the equation of the other tangent from the origin to this circle.
- c) If  $y = kx$  intersects the circle in two distinct points, state the range of values of  $k$ .

(3,5,2)

5. An ear-ring is to be made from silver wire and is designed in the shape of two touching circles with two tangents to the outer circle.



In the coordinate model the circles touch at  $(0,0)$ .

The inner circle has equation  $x^2 + y^2 - 5x = 0$ .

The outer circle intersects the x-axis at  $(8,0)$ .

The tangents meet the x-axis at  $(12,0)$ .

Find the total length of silver wire required to make this ear-ring.

(7)

6. The equation of a circle is  $x^2 + y^2 + 14x - 20y + 129 = 0$ .
- a) State the centre and radius of this circle.
- b) Verify that the point  $P(-3,12)$  lies on the circle and show that the equation of the tangent to the circle at  $P$  is  $y + 2x = 6$ .
- c) Show that the line  $y + 2x = 6$  is also a tangent to the circle  
 $x^2 + y^2 - 28x + 4y + 120 = 0$  and find the point of contact.

(2,5,5)

7. The line  $y - 2x = k$  is a tangent to the circle  $x^2 + y^2 - 6x - 2y + 5 = 0$ .

- a) Find two possible values of  $k$ .
- b) Find the coordinates of the two points where the line touches the circle.

(6,5)

## Problem Solving

### Answers

1. (a)  $x^2 + y^2 + 8x - 10y + 16 = 0$   
(b)  $(x - 8)^2 + (y + 4)^2 = 100$
2. i) (2,-1) ii) Q(-9,-1), P(8,7)  
iii)  $(x + 3)^2 + y^2 = 100$
3. (a)  $4y - 3x + 1 = 0$  (b) (-1,-1)  
(c)  $(x - 3)^2 + (y - 2)^2 = 25$
4. (a)  $2y + y - 11 = 0$   
(b) Roots are equal : Tangency
5. (a) (9,-5), 10, (-3,4), 5  
(b) Centres 15 units apart  
(c) (1,1) (d)  $3y = 4x - 1$
6. (a)  $y + x = 7$   
(b)  $x^2 + y^2 - 8x - 2y + 7 = 0$
7. (a) Substitute points into equation.  
(b) Roots equal : One root only. (6,2)
8. (a) Use equation of circle.  
(b)  $k = 5$  (c)  $4y = 3x + 21$
9. (a)  $(x - 1)^2 + (y - 3)^2 = 25$   
(b) Show  $PQ = kAB$   
(c) 4 units

## The Circle

## Problem Solving

## Sheet 2

1. (a) (-1,2) R = 5  
(b) P(-1,7) Q(4,2) M( $\frac{3}{2}, \frac{9}{2}$ )  
(c)  $(x + 1)^2 + (y - 2)^2 = 12.5$
2. (a) Proof  
(b)  $y = 3x - 29$   
(c)  $(x - 8)^2 + (y + 5)^2 = 170$
3.  $x^2 + y^2 - 6x - 12y + 20 = 0$  : (6,10) :  $4y + 3x = 58$
4. (a)  $x = 5$ , One root only  
(b)  $5y = 12x$   
(c)  $0 < k < \frac{12}{5}$
5. Total length =  $13\pi + 8\sqrt{3}$
6. (a) Centre (-7,10)  
(b) Proof  
(c) Equal roots at  $x = 6$ , (6,-6)
7. (a)  $k = 0$  or -10  
(b) (5,0) ; (5,8)